ADAPTIVE CONTROL OF MULTI-AGENT SYSTEMS FOR FINDING PEAKS OF UNKNOWN FIELDS

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ABSTRACT

In this paper, we consider a deterministic adaptive control framework to design and analyze a class of multi-agent systems that locate peaks of unknown static fields in a distributed and scalable manner. Each agent is driven by swarming and gradient ascent efforts based on its own recursively estimated field via locally collected measurements by itself and its neighboring agents. The convergence properties of the proposed multi-agent systems are analyzed. We also provide a sampling scheme to facilitate the convergence. The simulation study confirms the convergence analysis of the proposed algorithms.

1 Introduction

In recent years, significant enhancements have been made in the areas of sensor networks and mobile sensing agents [1–6]. Mobile sensing agents usually have an ad-hoc wireless communication network in which each agent usually shares information with neighboring agents within a short communication range, with limited memory and computational power. Mobile sensing agents are often spatially distributed in an uncertain surveillance environment. In order to achieve various tasks such as exploration, surveillance, and environmental monitoring, mobile sensing agents require distributed coordination to adapt to environments in order to perform a global goal.

Distributed control laws were proposed to achieve optimal coverage configurations for the known distributions [1]. Decentralized and adaptive control algorithms have been proposed in [6] for networks of robots to converge to optimal sensing configurations while simultaneously learning the distribution of sensory information in the environment.

Tanner [3] and Olfati-Saber [4] developed comprehensive analyses of the flocking algorithm by Reynolds [7]. In general, the collective swarm behaviors of birds/fish/ants/bees are known to be the outcomes of natural optimization [8, 9]. These flocking algorithms have been used to move mobile sensor networks in groups [10, 11].

Among other problems in mobile sensor networks, gradient climbing over an unknown field of interest has attracted much attention of control engineers [11–14]. The cooperative network of agents that performs adaptive gradient climbing in a distributed environment was presented in [12, 13]. The centralized network can adapt its configuration in response to the sensed environment in order to optimize its gradient climb.

In [11], a distributed learning and control algorithm is proposed to be executed by each agent independently to estimate an unknown field of interest from noisy measurements and to coordinate multiple agents in a distributed manner to discover peaks of the unknown field. Each mobile agent moves towards peaks of the field using the gradient of its estimated field while avoiding collision and maintaining communication connectivity. The convergence properties of the resulting collective stochastic algorithm were analyzed using the Ljung's ODE approach. In the analysis, the estimation error dynamics have been averaged out under sufficient conditions and so only the ODE of the controlled multi-agent system dynamics could be considered.

In this paper, we consider a deterministic adaptive control framework to design and analyze a class of multi-agent systems that locate peaks of unknown static fields in a distributed and

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scalable manner. We use swarming artificial potentials to maintain communication connectivity and avoid collisions. The proposed distributed adaptive control consists of swarming effort and the gradient-based motion control based on the recursively updated field. The associated recursive estimation laws have been developed by gradient-based and recursive least squares (RLS) algorithms. In contrast to [11], the closed-loop dynamics of the motion control and the parameter estimation for the multi-agent system under proposed strategies have been analyzed. A set of sufficient conditions for which the convergence of the closed-loop multi-agent system is achieved has been provided. To facilitate the successful convergence, we provide an additional scalable and distributed sampling strategy that keeps selective past measurements. The effectiveness of the proposed schemes has been demonstrated via the simulation study.

2 Preliminaries

In this section, we describe the mathematical framework for mobile sensing agent networks and explain notations used in this paper.

Notations are standard. Let $\mathbb{R}, \mathbb{R}_{\geq 0}, \mathbb{R}_{>0}$ denote, respectively, the set of real, non-negative real, and positive real. The positive definiteness (respectively, semi-definiteness) of a matrix A is denoted by $A \succ 0$ (respectively, $A \succeq 0$). $I_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix of size n. $\mathbf{1}_n \in \mathbb{R}^n$ denotes the column vector of size n whose elements are 1. $|N|$ denotes the cardinality of the set N. $\mathrm{diag}(A,B)$ denotes the (generalized) block diagonal matrix of $A \in \mathbb{R}^{a\times m}$, $B \in \mathbb{R}^{b\times n}$ and is defined by $\mathrm{diag}(A,B) = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \in \mathbb{R}^{(a+b)\times(m+n)}$.

We assume that n number of sensing agents are distributed over the surveillance region $Q \subset \mathbb{R}^2$. Q is assumed to be a convex and compact set. The identity of each agent is indexed by $I := \{1, 2, \cdots, n\}$. Let $q_i(t) \in Q$ be the location of the i-th sensing agent at time $t \in \mathbb{R}_{\geq 0}$ and we define $q := \mathrm{col}(q_1, q_2, \cdots, q_n) \in \mathbb{R}^{2n}$ the configuration of the multi-agent system.

To describe the group behavior of mobile sensing agents and interactions with neighbors with the limited communication capability, the graph notation is used. We assume that each agent can communicate with its neighboring agents within a limited transmission range, which is given by a radius of r. The neighborhood of agent i with a configuration of q is defined by $\mathcal{N}(i,q) := \{j \in I \,|\, (i,j) \in \mathcal{E}(q)\}$. Therefore, $(i,j) \in \mathcal{E}(q)$ if and only if $\|q_i(t) - q_j(t)\| \leq r$. We often use $\mathcal{N}_i$ instead of using $\mathcal{N}(i,q)$ for simplicity. We define $\overline{\mathcal{N}}_i$ as the union of index i and indices of its neighbors, i.e., $\overline{\mathcal{N}}_i := \{i\} \cup \mathcal{N}_i$. We use the adjacency matrix $A := [a_{ij}]$ of an undirected graph G as defined in [2]. $A := [a_{ij}]$ is symmetrical. The element a_{ij} of adjacency matrix is defined as $a_{ij} = \sigma_w(\varepsilon - d_{ij})$, with $\sigma_w(y) = \frac{1}{1+e^{-wy}}$, where d_{ij} is a distance between neighboring agent j and agent i itself, σ_w is the sigmoid function with constants $w > 0$ and $\varepsilon > 0$. The scalar graph Laplacian $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ is a matrix defined as $L := D(A) - A$, where $D(A)$ is a diagonal matrix given by, i.e., $D(A) := \mathrm{diag}(\sum_{j=1}^{n} a_{ij})$. The 2-dimensional graph Laplacian is defined as $\hat{L}_2 := L \otimes I_2$, where $\otimes$ is the Kronecker product. The quadratic disagreement function $\Psi_G : \mathbb{R}^{2n} \to \mathbb{R}_{\geq 0}$ is used to evaluate the group disagreement in the network of agents $\Psi_G(p) := \frac{1}{4}\sum_{(i,j)\in\mathcal{E}(q)} a_{ij}\|p_j - p_i\|^2$, where $p := \mathrm{col}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{2n}$. A disagreement function [4, 15] can be expressed via the Laplacian $\hat{L}_2$: $\Psi_G(p) = \frac{1}{2}p^T\hat{L}_2 p$, and hence the gradient of $\Psi_G(p)$ w.r.t. p is given by $\nabla\Psi_G(p) = \hat{L}_2 p$.

We use attractive and repulsive potential functions similar to ones used in [3, 4, 11] to make a swarming behavior. To enforce a group of agents to satisfy a set of algebraic constraints $\|q_i - q_j\| = d$ for all $j \in \mathcal{N}_i$, we use a smooth collective potential function [11]

$$\begin{aligned} U_1(q) &:= \sum_i \sum_{j\in\mathcal{N}(i,q), j\neq i} U_{ij}(\|q_i - q_j\|^2) \\ &= \sum_i \sum_{j\in\mathcal{N}(i,q), j\neq i} U_{ij}(r_{ij}), \end{aligned} \tag{1}$$

where $r_{ij} := \|q_i - q_j\|^2$. The pair-wise attractive/repulsive potential function $U_{ij}(\cdot)$ in (1) is defined by

$$U_{ij}(r_{ij}) := \frac{1}{2}\left(\log(\alpha + r_{ij}) + \frac{\alpha + d^2}{\alpha + r_{ij}}\right), \text{ if } r_{ij} < d_0^2,$$

otherwise (i.e., $r_{ij} \geq d_0^2$), it is defined according to the gradient of the potential, which will be described shortly.

Here $\alpha, d \in \mathbb{R}_{>0}$ and $d < d_0$. The gradient of the potential with respect to q_i for agent i is given by

$$\begin{aligned} \nabla U_1(q_i) &:= \frac{\partial U_1(q)}{\partial \tilde{q}_i}\Big|_{\tilde{q}_i = q_i} = \sum_{j\neq i} \frac{\partial U_{ij}(r)}{\partial r}\Big|_{r = r_{ij}} (q_i - q_j) \\ &= \begin{cases} \sum_{j\neq i} \frac{(r_{ij} - d^2)(q_i - q_j)}{(\alpha + r_{ij})^2} & \text{if } r_{ij} < d_0^2 \\ \sum_{j\neq i} \rho\left(\frac{\sqrt{r_{ij}} - d_0}{|d_1 - d_0|}\right) \frac{\|d_0^2 - d^2\|}{(\alpha + d_0^2)^2}(q_i - q_j) & \text{if } r_{ij} \geq d_0^2, \end{cases} \end{aligned}$$

where $\rho : \mathbb{R}_{\geq 0} \to [0,1]$ is the bump function [4]

$$\rho(z) := \begin{cases} 1, & z \in [0,h); \\ \frac{1}{2}\left[1 + \cos\left(\pi\frac{(z-h)}{(1-h)}\right)\right], & z \in [h,1]; \\ 0, & \text{otherwise.} \end{cases}$$

A potential U_2 [11] is also used to model the environment. U_2 enforces each agent to stay inside the closed and connected surveillance region in Q and prevents collisions with obstacles in Q. We construct U_2 such that it is radially unbounded in q, i.e., $U_2(q) \to \infty$ as $\|q\| \to \infty$. This condition will be used for making a Lyapunov function candidate radially unbounded. Define the total artificial potential by

$$U(q) := k_1U_1(q) + k_2U_2(q), \tag{2}$$

where $k_1, k_2 \in \mathbb{R}_{>0}$ are weighting factors.

3 Static environmental field modeling

Suppose that the scalar environmental field $\mu(\nu)$ is generated by a network of radial basis function [11]:

$$\mu(\nu) = \sum_{j=1}^{m} \phi_j(\nu)\theta^j = \phi^T(\nu)\theta, \tag{3}$$

where $\phi^T(\nu)$ and θ are defined respectively by

$$\begin{aligned} \phi^T(\nu) &= \begin{bmatrix} \phi_1(\nu) \ \phi_2(\nu) \ \cdots \ \phi_m(\nu) \end{bmatrix} \in \mathbb{R}^{1\times m}, \\ \theta &= \begin{bmatrix} \theta^1 \ \theta^2 \ \cdots \ \theta^m \end{bmatrix}^T \in \mathbb{R}^{m\times 1}. \end{aligned} \tag{4}$$

Gaussian radial basis functions $\phi_j(\nu)$ are given by

$$\phi_j(\nu) = \frac{1}{\beta_j} \exp\left(\frac{-\|\nu - \xi_j\|^2}{\sigma_j^2} \right), \forall j \in M, \tag{5}$$

where $M := \{1, \cdots, m\}$, σ_j is the width of the Gaussian basis and β_j is a normalizing constant. Centers of basis functions $\{\xi_j | j \in M\}$ are uniformly distributed in the surveillance region Q. The partial derivative of $\phi(x) \in \mathbb{R}^{m\times 1}$ with respect to $x \in \mathbb{R}^{2\times 1}$ evaluated at x^* is denoted by $\phi'(x^*)$ and is given as follows.

$$\phi'(x^*) := \frac{\partial \phi(x)}{\partial x}\Big|_{x=x^*} \in \mathbb{R}^{m\times 2}.$$

The gradient of the field at q_i is denoted by

$$\nabla\mu(q_i) = \frac{\partial \mu(x)}{\partial x}\Big|_{x=q_i} \in \mathbb{R}^{2\times 1}. \tag{6}$$

Using (3), (6) can be represented in terms of θ,

$$\nabla\mu(q_i) = \frac{\partial \phi^T(x)}{\partial x}\Big|_{x=q_i} \theta = \phi'^T(q_i)\theta \in \mathbb{R}^{2\times 1}. \tag{7}$$

The estimate of $\nabla\mu(q_i)$ based on $\hat{\theta}$ is denoted by $\nabla\hat{\mu}(q_i)$.

4 Distributed adaptive control

In this chapter, we propose a distributed adaptive control algorithm. The adaptive control law for each agent will be generated using only local information from neighboring agents. The dynamics of agent i is given by

$$\begin{aligned} \frac{dq_i(t)}{dt} &= p_i(t), \\ \frac{dp_i(t)}{dt} &= u_i(t), \end{aligned} \tag{8}$$

where $u_i(t)$ is the input of agent i. The control input $u_i(t)$ is proposed as follows.

$$\begin{aligned} u_i(t) = &- \nabla U(q_i(t)) - k_{d_i} p_i(t) - \nabla\Psi_G(p_i(t)) \\ &+ k_4 \phi'^T(q_i(t))\hat{\theta}_i(t), \end{aligned} \tag{9}$$

where $\hat{\theta}_i(t)$ is the estimate of $\theta(t)$ by agent i using the recursive parameter estimation algorithms.

To achieve a consensus, we use a quadratic disagreement function $\Psi_G(\hat{\theta}_d(t))$: $\mathbb{R}^{mn} \to \mathbb{R}_{\geq 0}$, which is defined as

$$\Psi_G(\hat{\theta}_d(t)) = \frac{1}{4} \sum_{(i,j)\in\varepsilon(q)} a_{ij} \parallel \hat{\theta}_j(t) - \hat{\theta}_i(t) \parallel^2 = \frac{1}{2}\hat{\theta}_d^T(t)\hat{L}_m\hat{\theta}_d(t),$$

where $\hat{L}_m = L \otimes I_m$, and $\hat{\theta}_d(t) = \mathrm{col}(\hat{\theta}_1(t), \cdots, \hat{\theta}_n(t))$. We have used its gradient at $\hat{\theta}_i(t)$ in (9), which is given by

$$\nabla\Psi_G(\hat{\theta}_i(t)) = \sum_{j\in\mathcal{N}_i} a_{ij}(q(t))(\hat{\theta}_i(t) - \hat{\theta}_j(t)).$$

To develop estimators for $\theta_i(t)$, the error vector $e_i(t)$ of agent i between the estimated values and measured values is defined by

$$e_i(t) = -\begin{bmatrix} \hat{\mu}_i(q_i(t)) - \mu(q_i(t)) \\ \hat{\mu}_i(q_j(t)) - \mu(q_j(t)) \\ \vdots \\ \hat{\mu}_i(q_k(t)) - \mu(q_k(t)) \end{bmatrix},$$

and can be rewritten by

$$e_i(t) = -\begin{bmatrix} \phi^T(q_i(t))\tilde{\theta}_i(t) \\ \phi^T(q_j(t))\tilde{\theta}_i(t) \\ \vdots \\ \phi^T(q_k(t))\tilde{\theta}_i(t) \end{bmatrix} = -\Phi_i(t)\tilde{\theta}_i(t),$$

where $\tilde{\theta}_i(t) := \hat{\theta}_i(t) - \theta_i(t)$ and

$$\Phi_i(t) = \begin{bmatrix} \phi^T(q_i(t)) \\ \phi^T(q_j(t)) \\ \vdots \\ \phi^T(q_k(t)) \end{bmatrix} \in \mathbb{R}^{|\overline{\mathcal{N}}_i| \times m},$$

with $j, \cdots, k \in \mathcal{N}_i$.

Using the gradient-based estimator, $\hat{\theta}_i(t)$ is updated by the following adaptive law.

$$\frac{d\hat{\theta}_i(t)}{dt} = \gamma_i \Phi_i^T(t) e_i(t) - \gamma_i k_4 \phi'(q_i(t)) p_i(t) - k_6 \gamma_i \nabla \Psi_G(\hat{\theta}_i(t)), \tag{10}$$

where γ_i is the estimation gain and k_6 is a consensus gain for parameter estimates.

Using the recursive least squares (RLS) estimator, $\hat{\theta}_i(t)$ is updated by the following adaptive law.

$$\begin{aligned} \frac{d\hat{\theta}_i(t)}{dt} =& P_i \Phi_i^T(t) e_i(t) - P_i k_4 \phi'(q_i(t)) p_i(t) \\ & - k_6 P_i \nabla \Psi_G(\hat{\theta}_i(t)), \\ \frac{dP_i(t)}{dt} =& - P_i(t) \Phi_i^T(t) \Phi_i(t) P_i(t), \end{aligned} \tag{11}$$

where $P_i(t)$ is defined by

$$P_i(t) = \left(\int_0^t \Phi_i^T(\tau) \Phi_i(\tau) d\tau \right)^{-1} \in \mathbb{R}^{m \times m}.$$

4.1 Collective dynamics of agents

The global collective cost function $C_d(q(t))$ for all agents is defined by $C_d(q(t)) = k_4 \sum_{i \in I} [\mu_{max} - \mu(q_i(t))] = k_4 \sum_{i \in I} [\mu_{max} - \phi^T(q_i(t))\theta]$. The collective estimate of $C_d(q(t))$ by all agents at time t is $\hat{C}_d(q(t))$ and is given by $\hat{C}_d(q(t)) = k_4 \sum_{i \in I} [\mu_{max} - \hat{\mu}_i(q_i(t))] = k_4 \sum_{i \in I} [\mu_{max} - \phi^T(q_i(t))\hat{\theta}_i(t)]$, where the estimate of $\mu(\nu)$ at ν in (3) by agent i is denoted by $\hat{\mu}_i(\nu)$ and is given as $\hat{\mu}_i(\nu) := \phi^T(\nu)\hat{\theta}_i(t)$. The gradient of C_d at $q(t)$ is given by

$$\nabla C_d(q(t)) = -k_4 \begin{bmatrix} \phi'^T(q_1(t)) \\ \phi'^T(q_2(t)) \\ \vdots \\ \phi'^T(q_n(t)) \end{bmatrix} \theta = -A_d(t)\theta_d,$$

where $\theta_d := \mathbf{1}_n \otimes \theta$. $A_d(t)$ is defined by

$$A_d(t) = k_4 \text{diag}(\phi'^T(q_1(t)), \cdots, \phi'^T(q_n(t))) \in \mathbb{R}^{2n \times mn}.$$

The collective estimate of $\nabla C_d(q(t))$ by all agents is denoted by $\nabla \hat{C}_d(q(t))$ and is given by

$$\nabla \hat{C}_d(q(t)) = -k_4 \begin{bmatrix} \phi'^T(q_1(t))\hat{\theta}_1(t) \\ \vdots \\ \phi'^T(q_n(t))\hat{\theta}_n(t) \end{bmatrix} = -A_d(t)\hat{\theta}_d(t).$$

The collective dynamics of agents from (8) and (9) are given by

$$\begin{aligned} \frac{dq(t)}{dt} = p(t), \quad \frac{dp(t)}{dt} &= -\nabla U(q(t)) - K_d p(t) \\ &- \nabla \Psi_G(p(t)) - \nabla \hat{C}_d(q(t)), \end{aligned} \tag{12}$$

where $q(t) = \text{col}(q_1(t), \cdots, q_n(t))$, and $p(t) = \text{col}(p_1(t), \cdots, p_n(t))$.

The collective version of the adaptive law in (10) for all agents is given by

$$\frac{d\hat{\theta}_d(t)}{dt} = \Gamma_d \Phi_d^T(t) e_d(t) - \Gamma_d A_d(t)^T p(t) - \Gamma_d \hat{L}_m \hat{\theta}_d(t), \tag{13}$$

where $\Gamma_d = \Gamma \otimes I_m$ and $\Gamma = \Gamma^T \succ 0$ is the diagonal matrix given by $\Gamma = \text{diag}(\gamma_1, \cdots, \gamma_n)$. Notice that $\hat{L}_m \hat{\theta}_d(t) = \hat{L}_m \tilde{\theta}_d(t)$ in (13). The collective error $e_d(t)$ is defined as

$$e_d(t) = \begin{bmatrix} e_1(t) \\ \vdots \\ e_n(t) \end{bmatrix} = \begin{bmatrix} -\Phi_1(t)\tilde{\theta}_1(t) \\ \vdots \\ -\Phi_n(t)\tilde{\theta}_n(t) \end{bmatrix} = -\Phi_d(t)\tilde{\theta}_d(t), \tag{14}$$

where $\tilde{\theta}_d(t) = \text{col}(\tilde{\theta}_1(t), \cdots, \tilde{\theta}_n(t))$ is the estimation error vector defined by $\tilde{\theta}_d(t) := \hat{\theta}_d(t) - \theta_d$. $\Phi_d(t) = \text{diag}(\Phi_1(t), \cdots, \Phi_n(t)) \in \mathbb{R}^{\left(\sum_{i \in I} |\overline{\mathcal{N}}_i|\right) \times mn}$.

The collective version of the adaptive law in (11) for all agents is given by

$$\begin{aligned} \frac{d\hat{\theta}_d(t)}{dt} =& - P_d(t) \Phi_d^T(t) \Phi_d(t) \tilde{\theta}_d(t) \\ & - P_d(t) A_d(t)^T p(t) - P_d(t) \hat{L}_m \hat{\theta}_d(t), \\ \frac{dP_d(t)}{dt} =& - P_d(t) \Phi_d^T(t) \Phi_d(t) P_d(t). \end{aligned} \tag{15}$$

where $P_d(t)$ is defined by $P_d(t) = \text{diag}(P_1(t), \cdots, P_n(t)) \in \mathbb{R}^{mn \times mn}$.

4.2 Convergence analysis

In this section, we present the results for convergence properties of the proposed multi-agent systems. To this end, we define the global performance cost function of the multi-agent system with the gradient-based algorithm in (13)

$$V_d(q(t),p(t),\tilde{\theta}_d(t)) = U(q(t)) + \frac{p^T(t)p(t)}{2} + C_d(q(t)) + \frac{\tilde{\theta}_d^T(t)\Gamma_d^{-1}\tilde{\theta}_d(t)}{2}. \tag{16}$$

The global performance cost function for the RLS algorithm in (15) are defined by

$$V_d(q(t),p(t),\tilde{\theta}_d(t)) = U(q(t)) + \frac{p^T(t)p(t)}{2} + C_d(q(t)) + \frac{\tilde{\theta}_d^T(t)P_d^{-1}(t)\tilde{\theta}_d(t)}{2}. \tag{17}$$

The collective performance cost function will be minimized by agents. The convergence properties of the multi-agent system is summarized by the following theorem.

Theorem 1. *We consider the distributed control law in* (9) *based on the gradient-based estimator in* (10) *(respectively, the RLS estimator in* (11)*) along with the global performance cost function V_d in* (16) *(respectively,* (17)*). For any initial state $x_0 = col(q_0, p_0, \tilde{\theta}_{d_0}) \in D_d$, where D_d is a compact set. Let $D_{Ad} = \{x \in D \mid V_d(x) \leq a\}$ be a level-set of the collective cost function. Let D_{cd} be the set of all points in D_{Ad}, where $\frac{dV_d(x)}{dt} = 0$. Then every solution starting from D_{Ad} approaches the largest invariant set M_d contained in D_{cd} as $t \to \infty$.*

Moreover, for the adaptive control using the gradient-based estimator (respectively, the RLS estimator), if $(\hat{L}_2 + K_d)$ and $(\Phi_d^T(t)\Phi_d(t) + \hat{L}_m)$ (respectively, $(\frac{1}{2}\Phi_d^T(t)\Phi_d(t) + \hat{L}_m)$) are positive definite, then any point $x^* = col(q^*, 0, 0)$ in M_d is a critical point of the cost function $V_d(x)$, which implies that x^* is either a (local) minimum of $V_d(x)$ or an inflection point, i.e., $\left.\frac{\partial V_d(x)}{\partial x}\right|_{x=x^*} = 0$, and $\hat{\theta}_d(t)$ converges to θ_d as $t \to \infty$.

Proof of Theorem 1 in the case of the gradient-based estimator.

Using (12), the time derivative of $V_d(x)$ in (16) is obtained by

$$\begin{aligned}\frac{dV_d}{dt} &= \begin{bmatrix} \nabla U(q(t)) + \nabla C_d(q(t)) \\ p(t) \end{bmatrix}^T \begin{bmatrix} p(t) \\ -\nabla U(q(t)) - \nabla \Psi_G(p(t)) - K_d p(t) - \nabla \hat{C}_d(q(t)) \end{bmatrix} \\ &\quad + \tilde{\theta}_d^T(t)\Gamma_d^{-1}\frac{d\hat{\theta}_d(t)}{dt} \\ &= -p^T(t)(\hat{L}_2 + K_d)p(t) \\ &\quad + \tilde{\theta}_d^T(t)\left(A_d^T(t)p(t) + \Gamma_d^{-1}\frac{d\hat{\theta}_d(t)}{dt}\right).\end{aligned} \tag{18}$$

With (13) and (18), we obtain

$$\begin{aligned}\frac{dV_d}{dt} &= -p^T(t)(\hat{L}_2 + K_d)p(t) \\ &\quad - \tilde{\theta}_d^T(t)(\Phi_d^T(t)\Phi_d(t) + \hat{L}_m)\tilde{\theta}_d(t) \leq 0.\end{aligned} \tag{19}$$

Let $x(t) = \mathrm{col}(q(t), p(t), \tilde{\theta}_d(t))$. From (16), we conclude that $V_d(x(t))$ is radially-unbounded, i.e., $V_d(x(t)) \to \infty$ as $\|x(t)\| \to \infty$. Hence, $D_{Ad} = \{x(t) \mid V_d(x(t)) \leq a\}$ is bounded and D_{Ad} with $\frac{dV_d(x)}{dt} \leq 0$ in (19) for all $x \in D_{Ad}$ is a positively invariant set. By LaSalle's invariant principle every point $x(t)$ in D_{Ad} approaches M_d included in D_{cd} which is given by

$$\begin{aligned}\Big\{x(t) \mid \frac{dV_d}{dt} &= -p^T(t)(\hat{L}_2 + K_d)p(t) \\ &\quad - \tilde{\theta}_d^T(t)(\Phi_d^T(t)\Phi_d(t) + \hat{L}_m)\tilde{\theta}_d(t) = 0\Big\},\end{aligned} \tag{20}$$

as $t \to \infty$. Let x^* be a solution that belongs to D_{cd}. If $(\hat{L}_2 + K_d) \succ 0$ and $(\Phi_d^T(t)\Phi_d(t) + \hat{L}_m) \succ 0$, $\forall x \in D_d$, from (20), any point x^* in M_d is the form of $x^*(t) = \mathrm{col}(q^*(t), p^* \equiv 0, \tilde{\theta}_d^* \equiv 0)$.

$$\tilde{\theta}_d^* \equiv 0 \Rightarrow 0 \equiv \hat{\theta}_d^*(t) - \theta_d \Rightarrow \nabla \hat{C}_d(q^*) \equiv \nabla C_d(q^*).$$

From (12), we have

$$\tilde{\theta}_d^* \equiv 0, p^*(t) \equiv 0 \Rightarrow \frac{dq^*(t)}{dt} \equiv 0 \Rightarrow 0 \equiv -\nabla U(q^*) - \nabla \hat{C}_d(q^*).$$

This implies that x^* is a critical point of the cost function $V_d(x)$ and $\hat{\theta}_d(t)$ converges to θ_d. QED.

Proof of Theorem 1 in the case of the RLS estimator.

Using (12), the time derivative of $V_d(x)$ in (17) is obtained by

$$\begin{aligned}\frac{dV_d}{dt} &= \begin{bmatrix} \nabla U(q(t)) + \nabla C_d(q(t)) \\ p(t) \end{bmatrix}^T \begin{bmatrix} p(t) \\ -\nabla U(q(t)) - \nabla \Psi_G(p(t)) - K_d p(t) - \nabla \hat{C}_d(q(t)) \end{bmatrix} \\ &\quad + \tilde{\theta}_d^T(t)P_d^{-1}(t)\frac{d\hat{\theta}_d(t)}{dt} + \frac{\tilde{\theta}_d^T(t)\frac{dP_d^{-1}}{dt}\tilde{\theta}_d(t)}{2} \\ &= -p^T(t)(\hat{L}_2 + K_d)p(t) \\ &\quad + \tilde{\theta}_d^T(t)\left(A_d^T(t)p(t) + P_d^{-1}(t)\frac{d\hat{\theta}_d(t)}{dt}\right) \\ &\quad + \frac{1}{2}\tilde{\theta}_d^T(t)\frac{dP_d^{-1}(t)}{dt}\tilde{\theta}_d(t).\end{aligned} \tag{21}$$

With (15) and (21), we obtain

$$\frac{dV_d}{dt} = -p^T(t)(\hat{L}_2 + K_d)p(t) - \tilde{\theta}_d^T(t)\left(\frac{1}{2}\Phi_d^T(t)\Phi_d(t) + \hat{L}_m\right)\tilde{\theta}_d(t) \le 0. \quad (22)$$

The rest of the proof follows as in the case of the gradient estimator. QED.

5 A sampling scheme for helping convergence

To improve the possibility of satisfying the sufficient conditions of convergence in Theorem 1, we provide a sampling scheme that will help on making $\Phi_d^T(t)\Phi_d(t)$ positive definite.

In this sampling scheme, at every sampling time t, a fixed number (m) of measurements sampled previously will be augmented to the fresh measurements available at time t for agent i. The selection of such m additional measurements at time t for agent i is as follows.

$$\bar{q}_{ij}(t) = \arg\min_{q\in\Omega_i(t)} \|q - \xi_j\|, \quad \forall j \in M, \quad (23)$$

where $M := \{1, \cdots, m\}$ and ξ_j is the center location of the j-th kernel. $\Omega_i(t)$ is defined by

$$\Omega_i(t) := \left(\bigcup_{k\in\bar{\mathcal{N}}_i(t)} \{\bar{q}_{ij}(t^-) \,|\, j \in M\}\right) \cup \left(\bigcup_{k\in\bar{\mathcal{N}}_i(t^-)} q_k(t^-)\right),$$

where t^- is the sampling time taken prior to t. Notice that this selection process in (23) is scalable and distributed.

Expanded $\Phi_i(t)$ due to the the augmented sampled data at time t is as follows.

$$\Phi_i(t) = \left[\,\phi(q_i(t)) \cdots \phi(\bar{q}_{i1}(t)) \cdots \phi(\bar{q}_{im}(t))\,\right]^T \in \mathbb{R}^{|\bar{\mathcal{N}}_i + m|\times m}.$$

6 Simulation results

We have applied the proposed multi-agent system to a scalar environmental field illustrated as a contour map in Fig 1. This static field was generated by the model in (3) with nine radial basis functions, i.e., $m = 9$. The estimated field was updated continuously and used for the coordination of each agent as proposed. Agents were launched at a set of randomly distributed initial positions, which was fixed and used for all simulations for fair comparison. Table 1 shows the parameters used for the numerical evaluation.

Mobile agents with transmission radius r of 0.5 were launched from initial positions to seek the peaks of the unknown static field in Fig 1. Figs. 1 and 2 show the simulated trajectories of agents with the gradient-based and RLS estimators, respectively. These trajectories show that agents with distributed control and learning algorithms successfully found peaks of the unknown field for both estimator cases. Notice also that some of agents would converge to the local minima of the field.

To evaluate the convergence rate of the parameter estimation for each agent, we compute the error norm $\|\tilde{\theta}_i(t)\|$, where $\tilde{\theta}_i(t) := \hat{\theta}_i(t) - \theta$ for agent i, and plot $\|\tilde{\theta}_i(t)\|$ for all agents $i \in I$ with respect to time t as in Figs. 3 and 4 for adaptive control with the gradient-based, and RLS estimators, respectively. Under this

Table 1. PARAMETERS IN THE SIMULATION

Parameters	Values
Number of agents n	20
Number of basis functions m	9
Surveillance region Q	$[0,3]^2$
(d, d_0, d_1, r)	$(0.3, 0.39, 0.5, 0.5)$
(k_1, k_2, k_4, k_6)	$(5, 1, 1, 1)$
k_d	$5I_{2n}$
$P(0) = \Gamma$	$0.5I_m$
$\theta(0)$	$0_{m\times 1}$

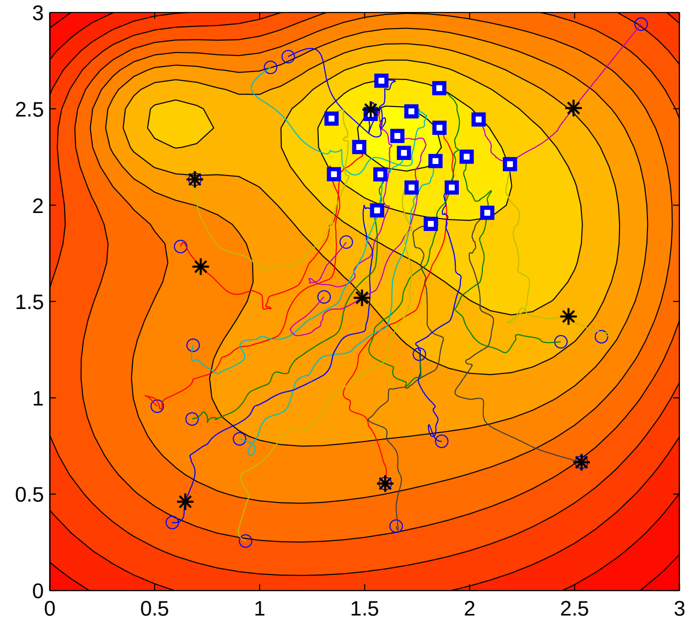

Figure 1. TRAJECTORIES OF AGENTS WITH THE DISTRIBUTED ADAPTIVE CONTROL BASED ON THE GRADIENT-BASED ESTIMATOR. THE INITIAL POSITIONS, THE ADDITIONAL SAMPLING POSITIONS, AND THE FINAL POSITIONS ARE MARKED BY CIRCLES (○), STARS (∗), AND SQUARES (□), RESPECTIVELY.

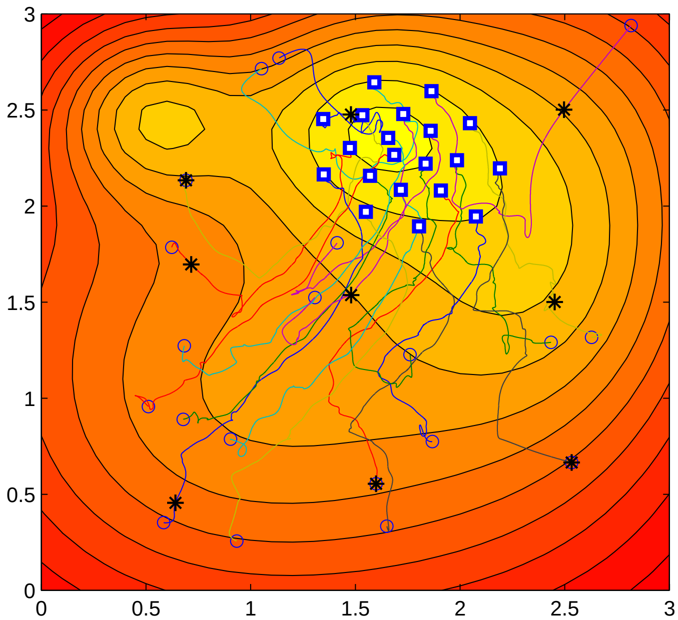

Figure 2. TRAJECTORIES OF AGENTS WITH THE DISTRIBUTED ADAPTIVE CONTROL BASED ON THE RLS ESTIMATOR. THE INITIAL POSITIONS, THE ADDITIONAL SAMPLING POSITIONS, AND THE FINAL POSITIONS ARE MARKED BY CIRCLES (○), STARS (∗), AND SQUARES (□), RESPECTIVELY.

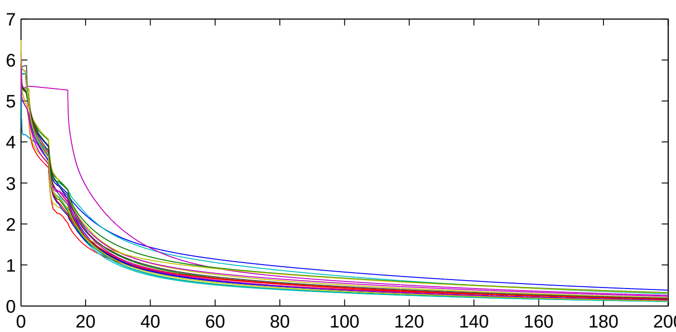

Figure 3. THE ERROR NORM $\|\tilde{\theta}_I(T)\|$ OF AGENT I W.R.T. TIME T FOR AGENTS WITH THE ADAPTIVE CONTROL WITH THE GRADIENT-BASED ESTIMATOR.

distributed control and learning algorithms, there are different groups of agents formed to share measurements locally. Therefore, the convergence rate of each agent depends on the initial configuration of the multi-agent system as shown in Figs. 3 and 4.

7 Conclusions

In this paper, we considered a deterministic adaptive control framework to design and analyze a class of multi-agent systems that locate peaks of unknown static fields of interest. Each agent was driven by swarming and gradient ascent efforts based on its own recursively estimated field via locally collected measurements. The convergence properties of the proposed multi-agent systems were analyzed. A sampling scheme to help the convergence was provided. The simulation study confirmed with the convergence analysis of the proposed algorithms. The future work is to consider the stochastic version of this problem taking into account the measurement noise.

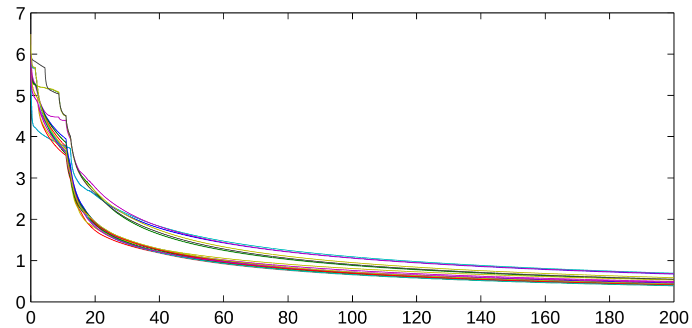

Figure 4. THE ERROR NORM $\|\tilde{\theta}_I(T)\|$ OF AGENT I W.R.T. TIME T FOR AGENTS WITH THE ADAPTIVE CONTROL WITH THE RLS ESTIMATOR.

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